ECE 520.651 Random Signal Analysis

Homework # 11

Due on Thursday, December 3, 2015.

Review the definitions and concepts in Sections IV.B, V.C and V.B before starting.

1. Solve problem IV.F.7 from Poor.
2. Solve problem IV.F.8 from Poor.
3. If $\Theta$ and $Y$ form a jointly Gaussian random vector (cf. example IV.B.3), with
   \[
   \mu = \begin{bmatrix} \mu_\Theta \\ \mu_Y \end{bmatrix} \quad \text{and covariance} \quad \Sigma = \begin{bmatrix} \Sigma_{\Theta\Theta} & \Sigma_{\Theta Y} \\ \Sigma_{Y\Theta} & \Sigma_{YY} \end{bmatrix},
   \]
   then show that the conditional density of $\Theta$ given $Y = y$ is also Gaussian, with
   \[
   \mu_{\Theta|Y} = \mu_{\Theta} + \Sigma_{\Theta Y} \Sigma_{YY}^{-1} (y - \mu_Y) \quad \text{and} \quad \Sigma_{\Theta|Y} = \Sigma_{\Theta\Theta} - \Sigma_{\Theta Y} \Sigma_{YY}^{-1} \Sigma_{Y\Theta}.
   \]
   Hint: In order to simplify the exponent in $w(\theta|y) = \frac{p(\theta,y)}{p(y)}$, you may want to compute $\Sigma^{-1}$ using the Schur complement method or Gaussian elimination.
   Conclude that the MMSE, MMAE and MAP estimate of $\Theta$ based on $Y$ are the same:
   \[
   \hat{\Theta}(Y) = \mu_{\Theta} + \Sigma_{\Theta Y} \Sigma_{YY}^{-1} (Y - \mu_Y),
   \]
   and the estimation error $e$ is a zero-mean random vector with (unconditional) covariance
   \[
   \Sigma_{ee} \equiv \mathbb{E} [e e^T] = \mathbb{E} [(\Theta - \hat{\Theta}(Y))(\Theta - \hat{\Theta}(Y))^T] = \Sigma_{\Theta\Theta} - \Sigma_{\Theta Y} \Sigma_{YY}^{-1} \Sigma_{Y\Theta}.
   \]
   NB: Here, $\Theta$ is a vector. Therefore the “squared error” is a matrix, as defined above.
4. Solve problem IV.F.24 from Poor.
5. Solve problem V.E.9 from Poor.

Work on additional problems related to the concepts in Sections V.B and V.C in preparation for the final exam. Do so in your study group to the extent possible, so that you may check each other’s work.