1. Write a two page summary (including any figures if you wish) of the article

Note: the copy available on-line through IEEE Xplore is missing one page of the article, and you may need to photocopy it from the MSE library.

2. Consider the HMM of Chapter 2, Figure 2.8, with state space $S = \{1, 2, 3\}$ and output alphabet $Y = \{0, 1\}$. In the notation of Chapter 2, let the transition probabilities for the output-producing transitions and null transitions be given, respectively, by

$$p(s'|s) \equiv \begin{bmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} \\ \frac{3}{4} & \frac{1}{4} & 0 \end{bmatrix} \quad \text{and} \quad q(s'|s) \equiv \begin{bmatrix} 0 & 0 & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}, \quad s, s' \in S.$$

Let the probabilities associated with the output producing transition from state $s$ to state $s'$ be given by

$$q(0|s \to s') \equiv \begin{bmatrix} 1 & \frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad q(1|s \to s') \equiv \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 1 & 1 & \frac{2}{3} \\ 1 & 1 & 1 \end{bmatrix},$$

and note that $q(1|s \to s')$ is simply $1 - q(0|s \to s')$. Let the initial state be $s_0 = 1$.

Perform the following calculations by hand, retaining intermediate answers in fractional form to preserve numerical accuracy.

(a) Draw a state diagram of this HMM, attaching state-labels to all nodes, probabilities $r(y,s'|s)$ or $q(s'|s)$ to all arcs and output-labels to all non-null arcs.

(b) Draw a 4-stage trellis for this HMM, showing only the paths which could have resulted in the output 0110.

(c) Compute the forward probabilities $\alpha_i(s)$ for the output sequence 0110, and indicate them on the trellis drawn above for (b).

(d) Compute the marginal probability of the output: $P(y_1y_2y_3y_4 = 0110|s_0 = 1)$. 

(e) Redraw the trellis with paths which could have given rise to 0110, and indicate on it the Viterbi probabilities $\gamma_i(s)$. Color the most like path given $y_1y_2y_3y_4 = 0110$.

(f) Redraw the trellis, showing paths which could have given rise to 0110, and compute the backward probabilities $\beta_i(s)$. Indicate them on this redrawn trellis.

(g) Compare $\beta_0(1)$ with the marginal probability $P(y_1y_2y_3y_4 = 0110|s_0 = 1)$ of (d).

(h) Calculate the a posteriori probabilities $P(t^i = t|y_1y_2y_3y_4 = 0110, s_0 = 1)$ for each arc in the trellis of (f). Show your answers on the trellis.

(i) Based on your calculations in (h), compute the expected counts $c(t)$ of each arc, and reestimate the HMM transition probability matrices $p(s'|s)$ and $q(s'|s)$. Caution: the matrix $p(s'|s)$ has only one entry for a non-null transition from $s$ to $s'$, while our trellis has up to $|\mathcal{Y}|$ distinct non-null arcs from $s$ to $s'$, one arc per output symbol. Make sure you consolidate their counts appropriately.

(j) Based on your calculations in (h), compute the expected counts $c(y,t)$ of each non-null arc, and reestimate the HMM emission probability matrix $q(0|s \to s')$.

Carefully review Chapter 2 from the Jelinek book after finishing the homework.