

050/520/600.666 Information Extraction from Speech and Text

Homework # 3

Due March 3, 2006.

1. Let X_1, \dots, X_n be a sequence of independently and identically distributed random variables taking values in a discrete and finite set \mathcal{X} , with common probability $P : \mathcal{X} \rightarrow [0, 1]$. Argue why the probability of observing a particular realization $X_1 = x_1, \dots, X_n = x_n$ is given by

$$\log P(x_1, \dots, x_n) = \log \prod_{t=1}^n P(x_t) = \sum_{x \in \mathcal{X}} N(x) \log P(x),$$

where $N(x)$ is the count of the symbol x in the sequence x_1, \dots, x_n .

Relate the formula derived above to Equation (16) in Chapter 4, page 68, of the textbook.

2. We discussed linear interpolation for smoothing a bigram language model in class, namely

$$P(w|v) = \lambda f(w|v) + (1 - \lambda) f(w),$$

where $f(\cdot|\cdot)$ and $f(\cdot)$ denote the appropriate relative frequency estimates, and λ is chosen so as to maximize the probability of some held-out data.

This problem considers a few alternative strategies for smoothing a bigram language model by directly modifying the *counts* observed in the training data. In particular, let $C(v, w)$ denote the count of the bigram $\langle v, w \rangle$ in the training text, and let $C^*(v, w)$ be the modified count. For some constant $\theta > 0$, consider the three cases

- (i) $C^*(v, w) = C(v, w) + \theta$,
- (ii) $C^*(v, w) = C(v, w) + \theta C(v)$, and
- (iii) $C^*(v, w) = C(v, w) + \theta C(v) f(w)$.

In each case, the smoothed bigram probability is calculated as

$$P^*(w|v) = \frac{C^*(v, w)}{\sum_{w' \in \mathcal{V}} C^*(v, w')} = \frac{C^*(v, w)}{C^*(v)}.$$

- (a) Let $N(v, w)$ denote the count of a bigram $\langle v, w \rangle$ in the held-out text. Compute, for (i)-(iii), an expression for the θ that maximizes the probability of the held-out text.

- (b) Show that if $N(v, w) = C(v, w)$ for all bigrams, then the optimal value is $\theta = 0$ in each of (i)-(iii). Why is this satisfactory?
- (c) Show that for each of (i)-(iii), P^* may be written as a linear interpolation. Identify the interpolation weight λ , and comment on the merits or drawbacks of the interpolation.

3. Consider the back-off bigram language model

$$P_{\text{BO}}(w|v) = \begin{cases} \frac{C(v,w) - \delta_1}{C(v)} & \text{if } C(v, w) > 0, \text{ and} \\ \alpha_1(v)f(w) & \text{otherwise} \end{cases}$$

where $\delta_1 \in (0, 1)$ is sometimes called a *constant discount* coefficient, $f(\cdot)$ denotes unigram relative frequencies and $\alpha_1(v)$, called the *back-off weight*, is chosen to make $P_{\text{BO}}(\cdot|v)$ a bona fide probability.

- (a) Develop an expression for $\alpha_1(v)$ in terms of the discount coefficient δ_1 , unigram probabilities $f(\cdot)$, and bigram counts $C(\cdot, \cdot)$.
- (b) Write a corresponding formula for replacing $f(w)$ with a back-off unigram model $P_{\text{BO}}(w)$ that uses a discount coefficient δ_0 and backs off to a *uniform* distribution on the entire vocabulary. Develop an expression for the back-off weight α_0 .
- (c) Does replacing $f(w)$ with $P_{\text{BO}}(w)$ necessitate recomputation of $\alpha_1(v)$? What does this say about the sequence in which back-off weights in an back-off N -gram model should be computed?