ECE 520.651 Random Signal Analysis

Homework # 10

Due on Tuesday, November 23, 2010.

Review Sections 6.8 from Stark & Woods and IV.C from Poor before starting the homework. The Poor problems below require concepts on pages 157-169.

1. Show that if $X[n]$ is a Martingale for $n = 0, 1, 2, \ldots$, then $Y[n] = X[n + m] - X[m]$ is also a Martingale. In other words, show that for every (fixed) $m \geq 0$,

$$E[Y[n] | Y[0], Y[1], \ldots, Y[n-1]] = Y[n-1].$$

Hint: $Y[0], \ldots, Y[n-1]$ do not completely determine $X[m], X[m+1], \ldots, X[m+n-1]$; but $Y[0], \ldots, Y[n-1]$ together with $X[m]$ do. Why?

2. Solve problem 6.46 from Stark and Woods. Refer to problem 6.45 for the definition of a random sequence $L[n]$ being “a Martingale w.r.t. another random sequence $X$.”

3. Solve problem IV.F.13(a) from Poor; for the same setting, compute the bias and variance of the estimator

$$\hat{\theta}(Y_1, \ldots, Y_n) = \frac{1}{n} \sum_{k=1}^{n} Y_k.$$

4. Solve problem IV.F.21(a) from Poor; note that the parameter being estimated is $\theta$, not $\lambda$, and $Y_1$ and $Y_2$ are therefore $\sim \text{Poisson}(-\log \theta)$.

5. Solve problem IV.F.22(a) from Poor.

Read the remainder of Section IV.C, i.e. Proposition IV.C.4 onwards, after finishing the homework, but before the next lecture.