ECE 520.651 Random Signal Analysis

Homework # 2

Due Tuesday, September 21, 2010.

Review Sections 10-15 of Prof. Papamarcou’s notes before starting the homework.

1. Given a $\sigma$-field $(\Omega, \mathcal{F})$ and an arbitrary $B \subseteq \Omega$, let

$$\tilde{\mathcal{F}} = \left\{ \tilde{A} : \tilde{A} = A \cap B \text{ for some } A \in \mathcal{F} \right\}.$$ 

i.e. $\tilde{\mathcal{F}}$ is a collection of subsets of $B$. Prove that $(B, \tilde{\mathcal{F}})$ is a $\sigma$-field.

Remark: $\tilde{\mathcal{F}}$ is called the restriction of $\mathcal{F}$ to $B$ or simply “$\mathcal{F}$ restricted to $B$.”

2. Let $P_1$ and $P_2$ be two different probability assignments on the same $\sigma$-field $(\Omega, \mathcal{F})$. Show that $(\Omega, \mathcal{F}, P_\lambda)$ is a valid probability space for any convex combination

$$P_\lambda(A) = \lambda P_1(A) + (1 - \lambda) P_2(A), \quad \forall A \in \mathcal{F}$$

of $P_1$ and $P_2$, where $0 \leq \lambda \leq 1$. i.e. show that $P_\lambda$ satisfies properties P1 through P3 on page 18 of Prof. Papamarcou’s notes.

3. Let $\Omega = (0, 1]$, $\mathcal{F} = \mathcal{B}((0, 1])$ and $P$ be the Lebesgue measure (see p25-26 in Prof. Papamarcou’s notes). Construct a random variable $X_1 : \Omega \rightarrow \mathbb{R}$ that takes the values $-1$, 0 and 1 with probability $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively.

4. On the same probability space, construct a random variable $X_2 : \Omega \rightarrow \mathbb{R}$ that is uniformly distributed in the interval $[-1, 1]$ and independent of $X_1$.

5. On the same probability space, construct a random variable $X_3 : \Omega \rightarrow \mathbb{R}$ that is uniformly distributed in the interval $[-1, 1]$ and not independent of $X_1$.

Read Chapter 2 from Stark and Woods after finishing the homework.