ECE 520.651 Random Signal Analysis

Homework # 3

Due 9:00 AM on Tuesday, September 29, 2009.

Review Chapter 4, Sections 4.1-4.2, from Stark and Woods and pages 47-60 from Prof. Papamarcou's notes before starting the homework.

1. Let $\Omega = [0,1)$, $\mathcal{F} = \mathcal{B}([0,1))$ and P be the Lebesgue measure. Determine whether the following *events* are mutually independent:

$$
\begin{aligned}
A &= \left[0, \frac{1}{2}\right) \\
B &= \left[0, \frac{1}{4}\right) \bigcup \left[\frac{1}{2}, \frac{3}{4}\right) \\
C &= \left[0, \frac{1}{8}\right) \bigcup \left[\frac{1}{4}, \frac{3}{8}\right) \bigcup \left[\frac{1}{2}, \frac{5}{8}\right) \bigcup \left[\frac{3}{4}, \frac{7}{8}\right)
\end{aligned}
$$

2. Does your construction of X_1 and X_2 in Homework #2, Problems 3 and 4 respectively, result in independent *random variables*? If not, redefine X_2 so that it still is uniformly distributed as required, but is also independent of X_1. Demonstrate that X_1 and X_2 are independent by computing the *conditional* CDF of X_2 given $X_1 = 0$, $X_1 = 1$, $X_1 = 2$ and $X_1 = 3$.

3. For any three events A, B and C, show that

$$P(A \cap C|B) = P(A|B) \times P(C|B) \quad \text{if and only if} \quad P(A|B \cap C) = P(A|B).$$

   In this case, we say that A and C are *conditionally independent* given B.

   (a) Is independence of A and C *sufficient* to make them conditionally independent?

   (b) Is independence *necessary* for them to be conditionally independent?

   If your answer is "yes," prove the assertion; if it is "no," provide a counterexample.

4. Solve problem **4.7** from Stark and Woods.

5. Solve problem **4.13** from Stark and Woods.

Start reading Chapter 3 from Stark and Woods after finishing the homework.