Review Chapter 4, Sections 4.1-4.2, from Stark and Woods and pages 47-60 from Prof. Papamarcou’s notes before starting the homework.

1. Let $\Omega = [0,1)$, $F = B([0,1))$ and $P$ be the Lebesgue measure. Determine whether the following events are mutually independent:

   $A = \left[0, \frac{1}{2}\right)$

   $B = \left[0, \frac{1}{4}\right) \cup \left[\frac{1}{2}, \frac{3}{4}\right)$

   $C = \left[0, \frac{1}{8}\right) \cup \left[\frac{1}{4}, \frac{3}{8}\right) \cup \left[\frac{1}{2}, \frac{5}{8}\right) \cup \left[\frac{3}{4}, \frac{7}{8}\right)$

2. Does your construction of $X_1$ and $X_2$ in Homework #2, Problems 3 and 4 respectively, result in independent random variables? If not, redefine $X_2$ so that it still is uniformly distributed as required, but is also independent of $X_1$. Demonstrate that $X_1$ and $X_2$ are independent by computing the conditional CDF of $X_2$ given $X_1 = 0, X_1 = 1, X_1 = 2$ and $X_1 = 3$.

3. For any three events $A$, $B$ and $C$, show that

   $$P(A \cap C|B) = P(A|B) \times P(C|B) \quad \text{if and only if} \quad P(A|B \cap C) = P(A|B).$$

   In this case, we say that $A$ and $C$ are conditionally independent given $B$.

   (a) Is independence of $A$ and $C$ sufficient to make them conditionally independent?

   (b) Is independence necessary for them to be conditionally independent?

   If your answer is “yes,” prove the assertion; if it is “no,” provide a counterexample.


Start reading Chapter 3 from Stark and Woods after finishing the homework.