ECE 520.651 Random Signal Analysis

Homework # 11

Due 9:00 AM on Tuesday, December 9, 2008.

Review Sections V.A through V.C from Poor and Sections 9.1 and 9.2 from Stark and Woods.

1. Solve problem **9.2** from Stark and Woods.

2. *Alternate proof of Theorem 9.1-4 from Stark and Woods.*

   (a) Let $\hat{X}_1$ be the LMMSE estimate of X_1 based on $\mathbf{Y} = [Y_a \ldots Y_b]^{\mathrm{T}}$, and $\hat{X}_2$ the LMMSE estimate of X_2 based on $\mathbf{Y}$. Use the *orthogonality principle* to show that the LMMSE estimate $\hat{Z}$ of
   $$Z = X_1 + X_2$$
   based on $\mathbf{Y}$ satisfies:
   $$\hat{Z} = \hat{X}_1 + \hat{X}_2 \qquad \text{almost surely.}$$
   Hint: Compute the difference
   $$E\left[\left(Z - \hat{Z}\right)^2\right] - E\left[\left(Z - (\hat{X}_1 + \hat{X}_2)\right)^2\right]$$
   and use the fact that linear combinations of any two (or all three) of $\hat{X}_1$, $\hat{X}_2$ and $\hat{Z}$ are in $\mathcal{H}_a^b$, the family of affine functions of $\mathbf{Y}$.

   (b) Let $\hat{X}_1$ be the LMMSE estimate of X based on $\mathbf{Y}_1 = [Y_a \ldots Y_b]^{\mathrm{T}}$, and $\hat{X}_2$ the LMMSE estimate of X based on $\mathbf{Y}_2 = [Y_c \ldots Y_d]^{\mathrm{T}}$. Use the *orthogonality principle* to show that if $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are *orthogonal* random vectors, then the LMMSE estimate $\hat{X}$ of X based on $\mathbf{Y} = \begin{bmatrix}\mathbf{Y}_1 & \mathbf{Y}_2\end{bmatrix}^T$ satisfies:
   $$\hat{X} = \hat{X}_1 + \hat{X}_2 \qquad \text{almost surely.}$$
   Hint: Assume all random variables are zero mean and, following (V.C.19) on p228 in the Poor book, write the LMMSE estimate of X based on $\mathbf{Y}$ as
   $$\hat{X} \;=\; \mathbf{h}^{\mathrm{T}}\mathbf{Y} \;=\; \begin{bmatrix}\mathbf{h}_1^{\mathrm{T}}\,\mathbf{h}_2^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{bmatrix} \;=\; \mathbf{h}_1^{\mathrm{T}}\mathbf{Y}_1 + \mathbf{h}_2^{\mathrm{T}}\mathbf{Y}_2,$$
   where $\mathbf{h} = \begin{bmatrix}\mathbf{h}_1\\ \mathbf{h}_2\end{bmatrix}$, $\mathbf{h}_1^{\mathrm{T}}\mathbf{Y}_1 \in \mathcal{H}_a^b$ and $\mathbf{h}_2^{\mathrm{T}}\mathbf{Y}_2 \in \mathcal{H}_c^d$. If $\mathbf{Y}_1 \perp \mathbf{Y}_2$, does it follow that the (scalar) random variables $\mathbf{h}_1^{\mathrm{T}}\mathbf{Y}_1$ and $\mathbf{h}_2^{\mathrm{T}}\mathbf{Y}_2$ are orthogonal for every $\mathbf{h}$?

3. Solve problem **V.E.2** from Poor.

4. Solve problem **V.E.5(a)** from Poor.

5. Solve problem **V.E.6** from Poor.