ECE 520.651 Random Signal Analysis

Homework # 10

Due 9:00 AM on Tuesday, December 2, 2008.

Review Chapter IV, Sections IV.A–D from Poor.

1. Solve problem **IV.F.5** from Poor.
2. Solve problem **IV.F.7** from Poor.
3. Solve problem **IV.F.8** from Poor.
4. Solve problem **IV.F.24(c)-(d)-(e)** from Poor.
5. Show that for jointly Gaussian random vectors $\mathbf{\Theta}$ and $\mathbf{Y}$, with

$$\mu_{\mathbf{\Theta Y}} = \begin{bmatrix} \mu_{\mathbf{\Theta}} \\ \mu_{\mathbf{Y}} \end{bmatrix} \qquad \text{and} \qquad \Sigma_{\mathbf{\Theta Y}} = \begin{bmatrix} \Sigma_{\mathbf{\Theta\Theta}} & \Sigma_{\mathbf{\Theta Y}} \\ \Sigma_{\mathbf{Y\Theta}} & \Sigma_{\mathbf{YY}} \end{bmatrix},$$

the conditional density of $\mathbf{\Theta}$ given $\mathbf{Y} = \mathbf{y}$ is also a Gaussian pdf, with

$$\mu_{\mathbf{\Theta}|\mathbf{Y}} = \mu_{\mathbf{\Theta}} + \Sigma_{\mathbf{\Theta Y}}\Sigma_{\mathbf{YY}}^{-1}(\mathbf{y} - \mu_{\mathbf{Y}}) \qquad \text{and} \qquad \Sigma_{\mathbf{\Theta}|\mathbf{Y}} = \Sigma_{\mathbf{\Theta\Theta}} - \Sigma_{\mathbf{\Theta Y}}\Sigma_{\mathbf{YY}}^{-1}\Sigma_{\mathbf{Y\Theta}}.$$

Conclude that the MMSE estimate of $\mathbf{\Theta}$ based on $\mathbf{Y}$ is the conditional mean

$$\widehat{\mathbf{\Theta}}(\mathbf{Y}) \;=\; \mu_{\mathbf{\Theta}} + \Sigma_{\mathbf{\Theta Y}}\Sigma_{\mathbf{YY}}^{-1}(\mathbf{Y} - \mu_{\mathbf{Y}}),$$

and the estimation error $\mathbf{e}$ is a $\mathbf{0}$-mean random vector with (unconditional) covariance

$$\Sigma_{\mathbf{ee}} \;\equiv\; E\left[\mathbf{e}\,\mathbf{e}^{\mathrm{T}}\right] \;=\; E\left[(\mathbf{\Theta} - \widehat{\mathbf{\Theta}}(\mathbf{Y}))(\mathbf{\Theta} - \widehat{\mathbf{\Theta}}(\mathbf{Y}))^{\mathrm{T}}\right] \;=\; \Sigma_{\mathbf{\Theta\Theta}} - \Sigma_{\mathbf{\Theta Y}}\Sigma_{\mathbf{YY}}^{-1}\Sigma_{\mathbf{Y\Theta}}.$$

Note that $\mathbf{\Theta}$ is a vector and the "squared error" is therefore a matrix, as defined above.

When reviewing the textbook, reflect upon the commentary that follows proofs of theorems, worked out problems, etc. Useful insight are often conveyed therein.