ECE 520.651 Random Signal Analysis

Homework # 9

Due 9:00 AM on Tuesday, November 25, 2008.

Carefully review the proofs of the *Factorization Theorem* (Proposition IV.C.1), the *Rao-Blackwell Theorem* (IV.C.2), the *Completeness Theorem* for Exponential Families (IV.C.3) and the *Consistency of the MLE* (IV.D.1), as well as the discussion of the conditions under which an estimator may achieve the information lower bound of Proposition IV.C.4.

1. Solve problem **IV.F.13(b)-(c)** from Poor. You may refer to answer(s) from HW#8.
2. Solve problem **IV.F.20** from Poor.
3. Solve problem **IV.F.21** from Poor.
4. Let $\theta \in (0, \infty)$ be a fixed but unknown parameter in a parametric family of pdf's

$$f_\theta(y) = \frac{1}{\theta} \exp\left\{-\frac{y}{\theta}\right\} u(y), \qquad y \in \mathbb{R},$$

   and let $Y_1^n \equiv Y_1, \ldots, Y_n$, be i.i.d. with common distribution $f_\theta(\cdot)$. Consider the problem of minimizing the squared error in the estimation of θ from Y_1^n.

   (a) Compute the Cramér-Rao lower bound for the variance of any unbiased estimator.

   (b) Compute the maximum likelihood estimate $\hat{\theta}_{\mathrm{ML}}(Y_1^n)$.

   (c) Show that $\hat{\theta}_{\mathrm{ML}}(Y_1^n)$ is the MVUE by computing its bias and variance.

   (d) Compare your answers in part (b), and the mean squared-error $E_\theta\left[(\hat{\theta}_{\mathrm{ML}} - \theta)^2\right]$, with the corresponding answers in example **IV.D.1**, and discuss the differences.

   (e) Compute the mean squared error of the estimator $\tilde{\theta}(Y_1^n) = \frac{1}{n+1}\sum_{k=1}^n Y_k$.

   (f) Discuss the bias vs variance trade-off using $\hat{\theta}_{\mathrm{ML}}(Y_1^n)$ and $\tilde{\theta}(Y_1^n)$ as examples.