

ECE 520.651 Random Signal Analysis

Homework # 2

Due 9:00 AM on Tuesday, September 23, 2008.

Review pages 1-50 of Prof. Papamarcou's notes before starting the homework.

1. Let P_1 and P_2 be two different probability assignments on the same σ -field (Ω, \mathcal{F}) . Show that $(\Omega, \mathcal{F}, P_\lambda)$ is a valid probability space for any convex combination

$$P_\lambda(A) = \lambda P_1(A) + (1 - \lambda)P_2(A), \quad \forall A \in \mathcal{F}$$

of P_1 and P_2 , where $0 \leq \lambda \leq 1$. i.e. show that P_λ satisfies properties P1 through P3 on page 18 of Prof. Papamarcou's notes.

2. Let $\Omega = (0, 1]$, $\mathcal{F} = \mathcal{B}((0, 1])$ and P be the Lebesgue measure (see p25-26 in Prof. Papamarcou's notes). Construct a random variable $X_1 : \Omega \rightarrow \mathbf{R}$ that takes four values, $\{0, 1, 2, 3\}$, with probability $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{8}$ respectively, and sketch the cumulative distribution function $F_{X_1}(\cdot)$ of X_1 .
3. On the same probability space, construct a random variable $X_2 : \Omega \rightarrow \mathbf{R}$ that is uniformly distributed in the interval $[0, 3]$, and sketch the cumulative distribution function $F_{X_2}(\cdot)$ of X_2 .
4. Is your construction of X_2 above such that X_1 and X_2 are *independent*? If not, redefine X_2 so that it still is uniformly distributed as required, but is also independent of X_1 . Demonstrate that X_1 and X_2 are independent by computing the *conditional* CDF of X_2 given $X_1 = 0$, $X_1 = 1$, $X_1 = 2$ and $X_1 = 3$.
5. For any mapping $f : \mathcal{X} \rightarrow \mathcal{Y}$, an arbitrary collection $\{A_i \subset \mathcal{X}, i \in I\}$, and an arbitrary collection $\{H_j \subset \mathcal{Y}, j \in J\}$, show that

$$\begin{aligned} f^{-1} \left(\bigcap_{j \in J} H_j \right) &= \bigcap_{j \in J} f^{-1}(H_j), \\ f \left(\bigcup_{i \in I} A_i \right) &= \bigcup_{i \in I} f(A_i) \quad \text{and} \\ f \left(\bigcap_{i \in I} A_i \right) &\subset \bigcap_{i \in I} f(A_i), \quad \text{but} \\ \text{in general, } f \left(\bigcap_{i \in I} A_i \right) &\neq \bigcap_{i \in I} f(A_i). \end{aligned}$$

Reread Chapter 2 from Stark and Woods after finishing the homework.