ECE 520.651 Random Signal Analysis

Homework # 1

Due 9:00 AM on Tuesday, September 16, 2008.

Read pages 1-22 from Prof. Papamarcou's notes before starting this homework.

1. Show that if a collection $\mathcal{F}$ of subsets of Ω is closed under complementation and *countable* unions, it is also closed under *countable* intersections.

2. Let A and B belong to some σ-field $\mathcal{F}$. Show that $\mathcal{F}$ contains the sets $A \setminus B$ and $A \triangle B$.

3. Let $\mathcal{F}$ be a σ-field of subsets of Ω and suppose that $B \in \mathcal{F}$. Show that $\mathcal{G} = \{A \cap B : A \in \mathcal{F}\}$ is a σ-field of subsets of B.

4. Show that the properties P1, P2, P5 and P9 togther *imply* the properties P1, P2 and P3 used to *define* a probability space.

5. Given a probability space $(\Omega, \mathcal{F}, P)$, and a decreasing sequence $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ of events in $\mathcal{F}$, show that
$$\lim_{n\to\infty} P(A_n) = P\left(\lim_{n\to\infty} A_n\right).$$
   This is property P10, which was stated in class without proof.

Read Chapters **1** and **2** from Stark and Woods after finishing this homework.