ECE 520.651 Random Signal Analysis

Homework # 11

Due 9:00 AM on Thursday, November 29, 2007.

Review Chapter IV, Sections A through D, from Poor.

1. Solve problem **IV.F.13(b)–(c)** from Poor; simply state your answer to part **(a)** from Homework #10, without repeating your derivation.
2. Solve problem **IV.F.17** from Poor.
3. Solve problem **IV.F.18** from Poor.
4. Solve problem **IV.F.21** from Poor.
5. Solve problem **IV.F.24** from Poor. Compare your answers in part (a), and the mean squared-error $E_\theta\left[(\hat{\theta}_{\mathrm{ML}} - \theta)^2\right]$, with the corresponding answers in example **IV.D.1**, and discuss the differences.

Review example **IV.B.3** and Chapter V, up to the statement of Proposition **V.B.1**.

6. Show that for jointly Gaussian random vectors $\mathbf{X}$ and $\mathbf{Y}$, with

$$\mu_{\mathbf{X},\mathbf{Y}} = \begin{bmatrix} \mu_{\mathbf{X}} \\ \mu_{\mathbf{Y}} \end{bmatrix} \qquad \text{and} \qquad \Sigma_{\mathbf{X},\mathbf{Y}} = \begin{bmatrix} \Sigma_{\mathbf{XX}} & \Sigma_{\mathbf{XY}} \\ \Sigma_{\mathbf{YX}} & \Sigma_{\mathbf{YY}} \end{bmatrix},$$

the conditional density of $\mathbf{X}$ given $\mathbf{Y} = \mathbf{y}$ is also a Gaussian pdf, with

$$\mu_{\mathbf{X}|\mathbf{Y}} = \mu_{\mathbf{X}} + \Sigma_{\mathbf{XY}}\Sigma_{\mathbf{YY}}^{-1}(\mathbf{y} - \mu_{\mathbf{Y}}) \qquad \text{and} \qquad \Sigma_{\mathbf{X}|\mathbf{Y}} = \Sigma_{\mathbf{XX}} - \Sigma_{\mathbf{XY}}\Sigma_{\mathbf{YY}}^{-1}\Sigma_{\mathbf{YX}}.$$

Conclude that the MMSE estimate of $\mathbf{X}$ based on $\mathbf{Y}$ is the conditional mean

$$\widehat{\mathbf{X}}(\mathbf{Y}) = \mu_{\mathbf{X}} + \Sigma_{\mathbf{XY}}\Sigma_{\mathbf{YY}}^{-1}(\mathbf{Y} - \mu_{\mathbf{Y}}),$$

and the estimation error $\mathbf{e}$ is a $\mathbf{0}$-mean random vector with (unconditional) covariance

$$\Sigma_{\mathbf{ee}} \equiv E\left[\mathbf{e}\,\mathbf{e}^{\mathrm{T}}\right] = E\left[(\mathbf{X} - \widehat{\mathbf{X}}(\mathbf{Y}))(\mathbf{X} - \widehat{\mathbf{X}}(\mathbf{Y}))^{\mathrm{T}}\right] = \Sigma_{\mathbf{XX}} - \Sigma_{\mathbf{XY}}\Sigma_{\mathbf{YY}}^{-1}\Sigma_{\mathbf{YX}}.$$