ECE 520.651 Random Signal Analysis

Homework # 2

Due 9:00 AM on Friday, September 21, 2007.

Review pages 1-50 of Prof. Papamarcou's notes before starting the homework.

1. Let P_1 and P_2 be two different probability assignments on the same σ-field $(\Omega,\, \mathcal{F})$. Show that $(\Omega,\, \mathcal{F},\, P_\lambda)$ is a valid probability space for any convex combination

$$P_\lambda(A) = \lambda P_1(A) + (1-\lambda)P_2(A), \qquad \forall\, A \in \mathcal{F}$$

of P_1 and P_2, where $0 \le \lambda \le 1$. i.e. show that P_λ satisfies properties P1 through P3 on page 18 of Prof. Papamarcou's notes.

2. Let $\Omega = (0, 1]$, $\mathcal{F} = \mathcal{B}((01,])$ and P be the Lebesgue measure (see p25-26 in Prof. Papamarcou's notes). Construct a random variable $X_1 : \Omega \to \boldsymbol{R}$ that takes four values, $\{0, 1, 2, 3\}$, with probability $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{8}$ respectively, and sketch the cumulative distribution function $F_{X_1}(\cdot)$ of X_1.

3. On the same probability space, construct a random variable $X_2 : \Omega \to \boldsymbol{R}$ that is uniformly distributed in the interval $[0, 3]$, and sketch the cumulative distribution function $F_{X_2}(\cdot)$ of X_2.

4. Is your construction of X_2 above such that X_1 and X_2 are *independent*? If not, redefine X_2 so that it still is uniformly distributed as required, but is also independent of X_1. Demonstrate that X_1 and X_2 are independent by computing the *conditional* CDF of X_2 given $X_1 = 0$, $X_1 = 1$, $X_1 = 2$ and $X_1 = 3$.

5. For any mapping $f : \mathcal{X} \to \mathcal{Y}$, an arbitrary collection $\{A_i \subset \mathcal{X},\, i \in I\}$, and an arbitrary collection $\{H_j \subset \mathcal{Y},\, j \in J\}$, show that

$$f^{-1}\left(\bigcap_{j\in J} H_j\right) = \bigcap_{j\in J} f^{-1}(H_j),$$

$$f\left(\bigcup_{i\in I} A_i\right) = \bigcup_{i\in I} f(A_i) \quad \text{and}$$

$$f\left(\bigcap_{i\in I} A_i\right) \subset \bigcap_{i\in I} f(A_i), \quad \text{but}$$

$$\text{in general,} \quad f\left(\bigcap_{i\in I} A_i\right) \neq \bigcap_{i\in I} f(A_i).$$

Reread Chapter 2 from Stark and Woods after finishing the homework.