ECE 520.651 Random Signal Analysis

Homework # 8

Due 9:00 AM on Friday, November 10, 2006.

Review Chapter IV, Sections IV.A and IV.B from Poor.

1. Solve problem **IV.F.3** from Poor.
2. Solve problem **IV.F.6** from Poor.
3. Solve problem **IV.F.7** from Poor.
4. Let $\Lambda = \mathbb{R}^+$, and consider the family of uniform pdfs $\left\{p_\theta(y) = \frac{1}{\theta}u(y)u(\theta - y),\, \theta \in \Lambda\right\}$. Show from first principles that if $Y_1, \ldots, Y_n$ are i.i.d with common density p_θ, then $T = T(Y_1, \ldots, Y_n) = \max\{Y_1, \ldots, Y_n\}$ is sufficient for estimating θ from $Y_1, \ldots, Y_n$. *i.e.* Show that the conditional pdf $p_\theta(y_1, \ldots, y_n \,|\, T = t)$ is not a function of θ.
5. Repeat the exercise above for $\Lambda = \mathbb{R}$, $\left\{p_\theta(y) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(y-\theta)^2}{2}},\, \theta \in \Lambda\right\}$ and $T = \sum_{i=1}^{n} Y_i$.

   Hint: One way to compute $p_\theta(y_1, \ldots, y_n \,|\, T = t)$ is to

   (a) first compute the pdf of $\begin{bmatrix} Y_1 & \ldots & Y_n & T \end{bmatrix}^{\mathrm{T}}$, a linear transform of $\begin{bmatrix} Y_1 & \ldots & Y_n \end{bmatrix}^{\mathrm{T}}$,

   (b) then compute the conditional pdf of the (sub)vector given T (cf. Example IV.B.3).