Review Chapter 7 from Cover and Thomas before starting the homework. Only problems 7.9 and 7.19 are mandatory. The rest are for practice; your answers will not be graded.

1. Problem 7.9: This problem uses the $Z$-channel from HW#7 to illustrate a key step in the proof of the channel coding theorem: identifying what values of $R$ are achievable if we “Fix $p(x)$.” at the beginning of the proof to a particular value. Here, it is $(\frac{1}{2}, \frac{1}{2})$.

2. Problem 7.13: This is another exercise in maximizing $I(X;Y)$.

3. Problem 7.15: This problem develops intuitions about joint typicality decoding through a concrete example.

   Compare for yourself the jointly typical set here with the typical set of the corresponding problem in Chapter 3.

4. Example 7.19: This problem exercises your ability to convert a “word problem” into a mathematically rigorous problem statement.

5. Problem 7.28: This problem introduces the notion of a sum channel.

   From this exercise, learn to think of $M = 2^C$ as the effective alphabet size of a channel, analogous to thinking of $2^H$ as the effective alphabet size of a source.

   Explain to yourself why, $\{\mathcal{X}, p(y|x), \mathcal{Y}\}$ is called the “sum” of its two component channels, $\{\mathcal{X}_1, p_1(y_1|x_1), \mathcal{Y}_1\}$ and $\{\mathcal{X}_2, p_2(y_2|x_2), \mathcal{Y}_2\}$, where

   $$\mathcal{X} = \mathcal{X}_1 \cup \mathcal{X}_2, \quad p(y|x) = \begin{cases} p_1(y|x) & \text{if } x \in \mathcal{X}_1, \\ 0 & \text{if } x \in \mathcal{X}_2 \end{cases}$$