1. Chapter 5, Problem 15
2. Chapter 5, Problem 17
3. Chapter 5, Problem 22
4. Chapter 5, Problem 25
5. Let $X$ be a random variable taking values on a set $\mathcal{X} = \{a, b, c, d, e, f\}$. The following exercise dwells on the fact that not only is the optimal code for $X$ not unique, even the set of codeword lengths for the optimal code(s) is not unique.

(a) Illustrate the construction of a binary Huffman code for $X$ which results in the code $C : \mathcal{X} \rightarrow \{0,1\}^*$ shown in the table below. (6 points)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
<th>Probability</th>
<th>$C(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$\frac{4}{12}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$b$</td>
<td>$\frac{2}{12}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$c$</td>
<td>$\frac{2}{12}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$d$</td>
<td>$\frac{2}{12}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$e$</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{12}$</td>
</tr>
<tr>
<td>$f$</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{12}$</td>
<td>0111</td>
</tr>
</tbody>
</table>

(b) Calculate the average codeword length $L(C)$ for $C$. (2 points)

(c) Calculate $L(C')$ for the code $C' : \mathcal{X} \rightarrow \{0,1\}^*$ given below, and confirm that $C'$ also minimizes the average codeword length for $X$. (2 points)
$\begin{array}{c|cc}
  x & P(x) & C'(x) \\
\hline
  a & \frac{4}{12} & 00 \\
  b & \frac{2}{12} & 010 \\
  c & \frac{2}{12} & 011 \\
  d & \frac{2}{12} & 10 \\
  e & \frac{1}{12} & 110 \\
  f & \frac{1}{12} & 111 \\
\end{array}$

(d) Draw the binary “code tree” for $C$ and $C'$. In each tree, label the node corresponding to the prefix 00 as $\alpha$, the node corresponding to the prefix 01 as $\beta$ and that corresponding to the prefix 1 as $\gamma$. (2 points)

(e) Explain the fact that $L(C) = L(C')$ by comparing the subtrees rooted at $\beta$ and $\gamma$ in the two code trees in (d). (4 points)

(f) The sorted set of codeword lengths for $C$ is (2,2,2,3,4,4); for $C'$ it is (2,2,3,3,3,3). Identify a third set of codeword lengths which yields an optimal code for $X$. (4 points)

Hint: Stare at the code trees of (d)!