ECE 520.447
Introduction to Information Theory and Coding

Homework #5
Due at 9:00 AM, October 15, 2002.

October 8, 2002

1. **Chapter 5, Problem 15**
2. **Chapter 5, Problem 17**
3. **Chapter 5, Problem 22**
4. **Chapter 5, Problem 25**
5. Let X be a random variable taking values on a set $\mathcal{X} = \{a, b, c, d, e, f\}$. The following exercise dwells on the fact that not only is the optimal *code* for X not unique, even the set of codeword *lengths* for the optimal code(s) is not unique.

   (a) Illustrate the construction of a binary Huffman code for X which results in the code $C : \mathcal{X} \to \{0, 1\}^*$ shown in the table below. (6 points)

x	$P(x)$	Probability					$C(x)$
a	$\frac{4}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	00
b	$\frac{2}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$		10
c	$\frac{2}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$			11
d	$\frac{2}{12}$	$\frac{}{12}$	$\frac{}{12}$				010
e	$\frac{1}{12}$	$\frac{}{12}$					0110
f	$\frac{1}{12}$						0111

   (b) Calculate the average codeword length $L(C)$ for C. (2 points)

   (c) Calculate $L(C')$ for the code $C' : \mathcal{X} \to \{0, 1\}^*$ given below, and confirm that C' also minimizes the average codeword length for X. (2 points)

x	$P(x)$	$C'(x)$
a	$\frac{4}{12}$	00
b	$\frac{2}{12}$	010
c	$\frac{2}{12}$	011
d	$\frac{2}{12}$	10
e	$\frac{1}{12}$	110
f	$\frac{1}{12}$	111

(d) Draw the binary "code tree" for C and C'. In each tree, label the node corresponding to the prefix 00 as α, the node corresponding to the prefix 01 as β and that corresponding to the prefix 1 as γ. (2 points)

(e) Explain the fact that $L(C) = L(C')$ by comparing the subtrees rooted at β and γ in the two code trees in (d). (4 points)

(f) The *sorted set of codeword lengths* for C is (2,2,2,3,4,4); for C' it is (2,2,3,3,3,3). Identify a third *set of codeword lengths* which yields an optimal code for X. (4 points)

Hint: Stare at the code trees of (d)!