Intrinsic Fourier Analysis on the Manifold of Speech Sounds

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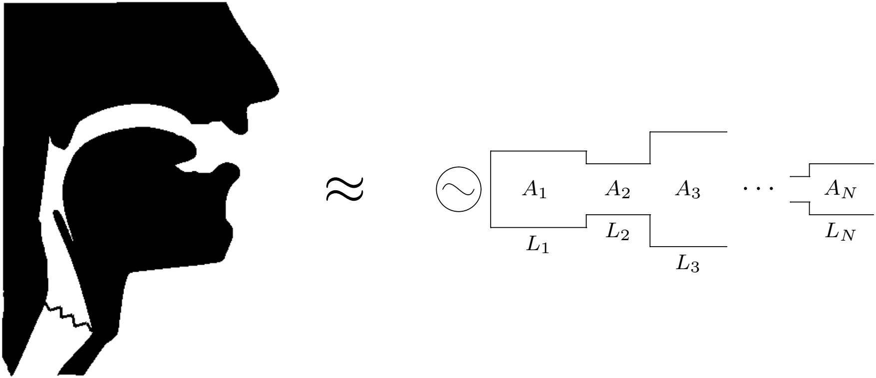
ICASSP 2006

Objectives

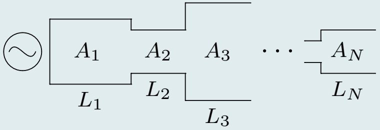
1. Motivate the existence of a low-dimensional manifold structure for speech
2. Present an algorithm to compute the intrinsic spectrogram

The Physics of Speech Production



The Physics of Speech Production

Acoustic Tube Model

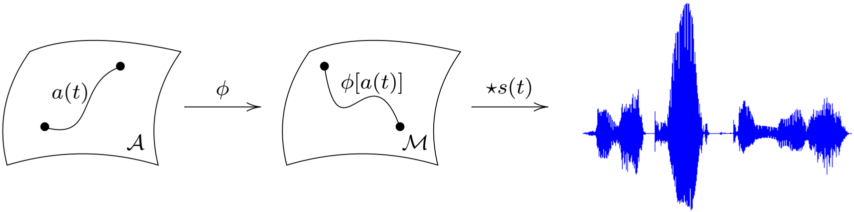


$$g(\omega) = \frac{\mathbf{Z}_r(\omega)}{M_{22} - \mathbf{Z}_r(\omega) M_{12}}$$

$$M = \prod_{i=1}^{N} \begin{bmatrix} \cos \frac{\omega L_i}{c} & i \frac{A_i}{\rho_0 c} \sin \frac{\omega L_i}{c} \\ i \frac{\rho_0 c}{A_i} \sin \frac{\omega L_i}{c} & \cos \frac{\omega L_i}{c} \end{bmatrix}$$

- The map $\phi_N : \{L_i\} \times \{A_i\} \rightarrow \mathcal{M}_N$ is a diffeomorphism
- Inverse map ϕ^{-1} is a coordinate chart on $\mathcal{M}_N$
- $\mathcal{M}_N$ is a smooth, $2N$-dimensional submanifold of $\mathcal{L}^2$
- $\mathcal{M}_N$ is extrinsically curved and spans the ambient space

The Speech Manifold



- $\mathcal{A} =$ set of vocal tract articulatory configurations
- $\mathcal{M} =$ set of vocal tract transfer functions
- Physics $\Rightarrow \phi : \mathcal{A} \to \mathcal{M}$ is a diffeomorphism
- Low $\dim(\mathcal{A}) \Rightarrow \mathcal{M}$ is a low-dimensional manifold

Intrinsic Spectrogram Representation

- For a signal $x(t)$, let

$$\vec{x}_i = i^{\text{th}} \text{ signal window}$$
$$\vec{y}_i = \|\text{DFT}(\vec{x}_i)\| \in \mathcal{M} \subset \mathbb{R}^H$$

- Traditional spectrogram, $S(t_i, \omega_j) = \vec{y}_i[j]$
- Rewrite: $S(t_i, \omega_j) = f_j(\vec{y}_i)$ where $f_j : \mathbb{R}^H \to \mathbb{R}$, $f_j(\vec{v}) = \vec{v}[j]$

Our Goal

Implement intrinsic projection maps

The Laplacian Operator, $\Delta_{\mathcal{M}}$

- Second-order differential operator on manifold $\mathcal{M}$
- Normalized eigenfunctions $\{e_i\}$ form orthogonal basis for $\mathcal{L}^2(\mathcal{M})$ (*i.e.* $f = \sum_i a_i e_i$)
- Define smoothness functional:

$$S[f] = \int_{\mathcal{M}} \|\nabla_{\mathcal{M}} f\|^2 d\mu = \langle \Delta_{\mathcal{M}} f, f \rangle_{\mathcal{L}^2(\mathcal{M})}$$

$$S[e_i] = \lambda_i$$

- Low $\lambda_i \Rightarrow e_i$ varies smoother with geodesic distance along manifold

The Graph Laplacian Operator, L_G

- Given $x_1, x_2, \ldots x_N \in \mathcal{M}$ construct k-nearest neighbor adjacency graph G, with adjacency matrix W
- $L_G = W - D$. where $D_{ii} = \sum_j W_{ij}$
- Analogous to $\Delta_{\mathcal{M}}$, but restricted to functions on graph
- $S_G[\mathbf{f}] = \mathbf{f}^T L_G \mathbf{f}$, where $\mathbf{f} = \langle f(x_1), ..., f(x_N)\rangle^T$

Computing Intrinsic Maps

- Solve optimization problem:

$$f^* = \arg\min_{f \in \mathcal{H}_K} \|f\|_K^2 + \xi \mathbf{f}^T L \mathbf{f}$$

- Admits solutions of form:

$$f_j^*(v) = \sum_{i=1}^{N} \alpha_i^j K(x_i, v)$$

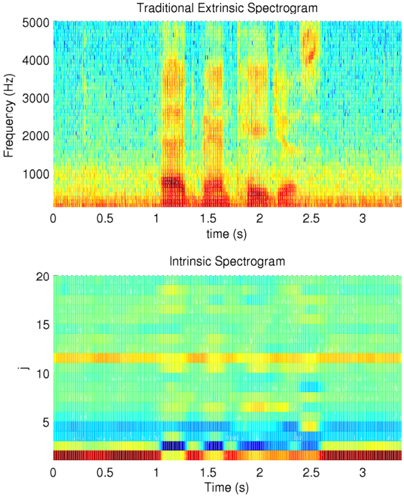
where

- $\alpha^j \in \mathbb{R}^N$ is the j-th eigenvector to $(\xi I + LK)\alpha = \lambda K \alpha$
- K is the $N \times N$ Gram matrix with $K_{ij} = K(x_i, x_j)$

Algorithm Summary

1. Supply a large set of Fourier amplitude spectra, $\{x_i\}$, across all phonetic classes
2. Calculate the graph Laplacian over $\{x_i\}$
3. Solve the optimization problem to recover intrinsic basis projection maps
4. Project traditional spectrogram on the intrinsic basis

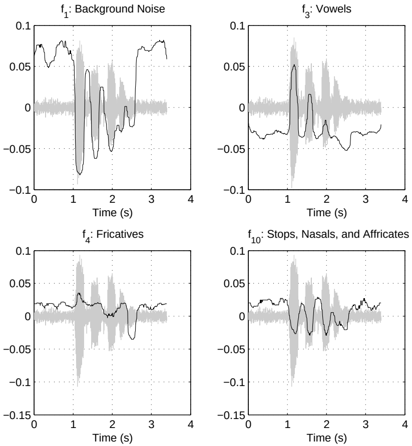
An Example: “advantageous”



- $\{x_1, \ldots, x_N\}$: 10 examples of 58 TIMIT phonemes
- $k = 6$, $\xi = 1$, $K(x, y) = x^T y$
- Activity limited to first ten components

Efficient information encoding

Instrinsic Component Behavior

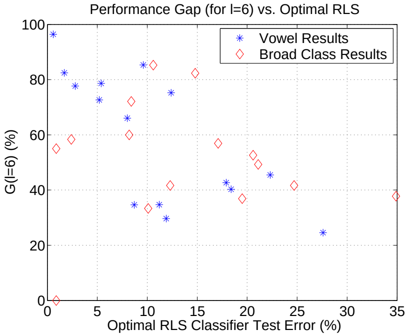


- Initial components provide broad class distinctions
- Higher λ components differentiate smaller nuances

Semi-supervised Classification

- Given l labelled and u unlabelled examples
- Construct intrinsic basis with u unlabelled examples
- $E(l) =$ *extrinsic* test error
  $I(l) =$ *intrinsic* test error
  $O(u+l) =$ *optimal* test error
- Performance Gap Improvement:

$$G(l) = \frac{E(l) - I(l)}{E(l) - O}$$



Conclusions

- Speech sounds have an underlying low-dimensional manifold structure
- Manifold structure can be exploited for novel speech representations
- Intrinsic Fourier analysis provides a compact representation for speech signals
- Approach shows promise in speech recognition applications