# ECE 520.651 Random Signal Analysis

Homework # 4

Due 9:00 AM on Friday, October 6, 2006.

Review Chapters 3 and 4 from Stark and Woods (again) before starting the homework.

1. Solve problem **3.28** from Stark and Woods.

2. Solve problem **4.31** from Stark and Woods. (Solve and use **4.30** if you wish.)

3. Solve problem **4.45** from Stark and Woods. (The Chi-square r.v. is defined in **4.41**.)

4. Prove that convergence *in probability* implies convergence *in distribution*. [*Hint*: First show that for any two random variables $X$ and $Y$,

$$P(X \leq x - \epsilon) \;\leq\; P(Y \leq x) + P(|Y - X| \geq \epsilon), \qquad \forall\, x \in \mathbb{R},\, \epsilon > 0,$$

and

$$P(X > x + \epsilon) \;\leq\; P(Y > x) + P(|Y - X| \geq \epsilon), \qquad \forall\, x \in \mathbb{R},\, \epsilon > 0.$$

Use the inequalities to show that if $X_n \to X$ in probability then, at all points $x$ where $F_X(x)$ is continuous, the sequence of *points* $F_{X_n}(x)$ must converge to the value $F_X(x)$.]

Where does your proof fail if $x$ is a point at which $F_X(x)$ has a discontinuity?

5. We have shown in class that

   (a) $X_n \to X$ in probability $\quad\Rightarrow\quad$ $X_n \to X$ in distribution, and

   (b) $X_n \to X$ in $r$-th mean $\quad\Rightarrow\quad$ $X_n \to X$ in probability,

   but the converse is not *generally* true in either case. This problem investigates *special cases* in which the converse *is* true.

   (c) Show that if $X_n \to c$ in distribution, where $c$ is a constant, then $X_n \to c$ in probability. In other words, if

$$\lim_{n\to\infty} F_{X_n}(x) = u(x - c) \quad \forall\, x \neq c \qquad \Rightarrow \qquad \lim_{n\to\infty} P\left(|X_n - c| > \epsilon\right) = 0 \quad \forall\, \epsilon > 0,$$

   where $u(\cdot)$ is the unit step function.

(d) Show that if $X_n \to X$ in probability and the $X_n$'s are uniformly bounded, then $X_n \to X$ in $r$-th mean. In other words, for some constant $c < \infty$, if

$$P(|X_n| \leq c) = 1 \quad \forall\, n,$$

then show that

$$\lim_{n\to\infty} P(|X_n - X| > \epsilon) = 0 \quad \forall\, \epsilon > 0 \qquad \Rightarrow \qquad \lim_{n\to\infty} E(|X_n - X|^r) = 0 \quad \forall\, \epsilon > 0.$$

Verify for yourself (no need to write up this part) that the counterexamples to the general converse of (a) and (b) that were presented in class do not satisfy the stronger conditions under which the implications (c) and (d) hold.

Read pages 63-81 from Prof. Papamarcou's notes and Section 6.7 from Stark and Woods.